Ruby Mishra, T.K.Naskar, Sanjib Acharya / International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 3, Issue 1, January -February 2013, pp.1193-1199

Synthesis of coupler curve of a Four Bar Linkage with joint Clearances

Ruby Mishra[*], T.K.Naskar[**] and Sanjib Acharya[**]

[*] School of Mechanical Engineering, KIIT University, BBSR- 751024, India
[**] Mechanical Engineering Departments, Jadavpur University, Kolkata-700032, India

Abstract

In this paper, synthesis of linkages with joint clearances to generate a desired coupler curve is studied. These clearances are treated as a mass less link and the equation of its motion were found by using Lagrange's equation. Dynamic parameters are used in Lagrange's equation to define the direction of motion of clearance. Deferential evaluation (DE) method was used to analyze the optimization of link parameters for minimizing the error between desired and actual path due to clearances at joint of coupler and follower. .

Key words: path synthesis, joint clearances, Lagrange's equation, Differential evaluation (DE), optimization.

1. INTODUCTION

Joint clearance is a important factor that affect the dynamic stability and the performance of mechanical system. It is generally exist in all kinds of planar mechanism. Clearance can causes vibration and noise and also affect the dynamic properties of the machine. So clearance may play a important role to define the kinematic and dynamic behavior of mechanisms. Without considering clearance we assume the linkage is called rigid link and when clearance is assumed in joints, the resulting linkage is called flexible linkage. A no. of works is reported in journals on flexible linkages and modeling of joint clearance.

The initial framework for the study of planar flexible link mechanisms has been provided by Boronkay and Mei [1] analyzed linkages whose pivots were replaced by flexible joints. Dubowsky [2] investigated the effects of joint clearance in a slider crank mechanism considering impact model. Burns and Crossley [3] in their investigation of the structural permutations of flexible link 4-bar chains having revolute pairs. This led to the development of techniques for the dimensional synthesis of devices that satisfied specific motion requirements by undergoing large elastic deformations. T. Furuhashi, N. Morita and M. Matsuura [4-5] determined angular directions of joint clearances by assuming continuous contact in revolute joints with clearance and using Lagrange function. Ting et al [6] presented an approach to identify the position and direction errors due to the joint clearance of linkages and manipulators. Joint clearance was modeled like a small link with length equals to one half of the clearance. A geometrical model was used in their method to assess the output position or direction variation, to predict the limit of position uncertainty and to determine the maximum clearance. Mallik and Dhande [7] introduced a stochastic model of the four-bar, path-generating linkage with tolerance and clearance to analyze the mechanical error in the path of a coupler point. Tolerances and clearances were assumed to be random variables. Schwab et al [8] studied dynamic response of mechanisms and machines affected by clearance in revolute joint and a comparison was made between several continuous contact force models and an impact model. Tsai and Lai [9] performed position analysis of a planar 4-bar mechanism with joint clearance by using loop-closure equations. Clearance was treated as a virtual link in this work. Flores and Ambrosio et al [10] performed dynamic analysis of mechanical systems considering realistic joint characteristics like joints with clearance and lubrication. The work analyzed contact impact forces of the kinematics and dynamic system. The joint clearance was modeled as a contact pair with dry contact as well as lubrication. For the lubricated case, the theory of hydrodynamic journal–bearings was used to compute the forces generated by lubrication action. Tsai and Lai [11] used equivalent kinematical pair in multi-loop linkage to model the motion freedoms obtained from joint clearances. The clearances were considered to be virtual links. Erkaya and Uzmay [12] studied the effects of joint clearance on quality of path generation and transmission angle. In the paper GA approach was used to determine the direction of joint clearance relative to input link of a 4-bar linkage.

In this paper, the nonlinear behavior in a four bar mechanism with joint clearance at the coupler and follower and its effect on path generation is analyzed. Differential evaluation method using langrage's equation is applied to describe the direction of joint clearances. And also using this method an objective function is defined and parametric relations between input variable and direction of joint clearances are considered as the constraints for solving nonlinear differential equation .Using DE optimization of the mechanism link parameters is also performed to minimize the path errors between desired and actual mechanisms is analyzed.

2. Modeling of Joint clearance

It is assumed that links of mechanism are connected to each other by revolute joints with clearance. Joint clearance, shown in Fig 1.Joint clearance r_2 is defined as the difference between the radii of the pin and hole, r_B and r_J respectively. When the journal gives impact on the bearing wall, normal and tangential force occurs. Also, if the friction is negligible, the direction of joint clearance vector coincides with the direction of normal force at the contact point. When the continuous contact mode between journal and bearing at joint is considered, the clearance may be modeled as vector which corresponds to mass less virtual link with the length equal to joint clearance.
The equivalent clearance (clearance vector) can be defined in the form

$$r_2 = r_B - r_J \tag{1}$$

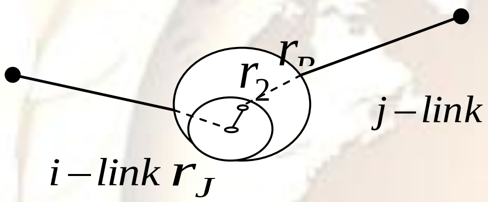

Fig.1. Equivalent clearance link

3. Synthesis of four bar path generator with joint clearance.

Dimensional synthesis of 4-bar linkage for path generation involves

A) The determination of the dimension of the linkage related to the desired path
B) The minimization of structural error which is subject to a set of size and geometric constant.

3.1. 4-Bar mechanism with double clearance

A four-bar mechanism with double clearance, as shown in Fig (2), is considered as an example to determine the effect of joint clearance between crank and coupler on path generation.
Kinematic equations are derived from the vector configuration of the mechanism, shown in Fig (3)

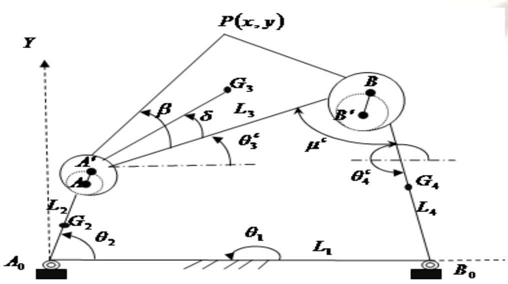

Fig.2. Schematic representation of a 4-bar mechanism with clearance

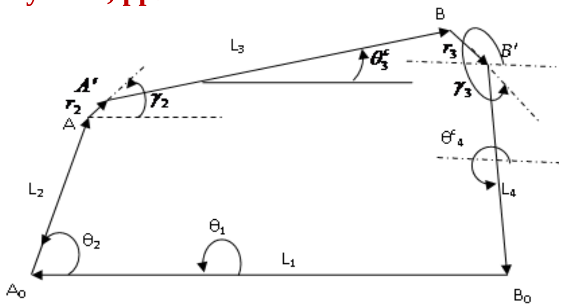

Fig. 3 .vector representation of mechanism

From the close loop vector relation

$$L_1 e^{i\theta_1} + L_2 e^{i\theta_2} + L_3 e^{i\theta_3^c} + L_4 e^{i\theta_4^c} + r_2 e^{i\gamma_2} + r_3 e^{i\gamma_3} = 0 \tag{2}$$

By separating Eq. (2), into its real part and imaginary part and using trigonometric relations, θ_3^c *and* θ_4^c with joint clearance can be expressed as a function of θ_2 and γ_2 respectively:

$$\theta_3^c = 2\tan^{-1}\left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right) \tag{3}$$

$$\theta_4^c = \cos^{-1}\left[\frac{L_1 - L_2\cos\theta_2 - r_2\cos\gamma_2 - L_3\cos\theta_3^c - r_3\cos\gamma_3}{L_4}\right] \ldots(4)$$

Where the superscript c denotes the value with clearance. A, B, C terms are given, with joint clearance respectively .

$$\begin{aligned} A = &-(L_1 + L_3)(2L_2\cos\theta_2 + 2r_2\cos\gamma_2) + 2r_3\cos\gamma_3 \\ &+ 2L_2 r_2\cos(\theta_2 - \gamma_2) + 2L_2 r_3\cos(\theta_2 - \gamma_3) + 2r_2 r_3\cos(\gamma_2 - \gamma_3) \ldots(5) \\ &+ 2L_1 L_3 + L_1^2 + L_2^2 + L_3^2 - L_4^2 + r_2^2 + r_3^2 \end{aligned}$$

$$B = 4L_3(L_2\sin\theta_2 + r_2\sin\gamma_2 + r_3\sin\gamma_3) \ldots(6)$$

$$\begin{aligned} C = &(L_3 - L_1)2L_2\cos\theta_2 + 2r_2\cos\gamma_2 + 2r_3\cos\gamma_3 + \\ &2L_2 r_2\cos(\theta_2 - \gamma_2) + 2L_2 r_3\cos(\theta_2 - \gamma_3) + 2r_2 r_3\cos(\gamma_2 - \gamma_3) - 2L_1 L_3 \\ &+ L_1^2 + L_2^2 + L_3^2 - L_4^2 + r_2^2 + r_3^2 \end{aligned} \tag{7}$$

As shown in Fig.2, the position of the coupler point $P(x, y)$ relative to the crank pivot A_1 is given with joint clearance, respectively:

$$P_x^c = L_2\cos(\theta_2) + r_2\cos\gamma_2 + A'P\cos(\theta_3^c + \beta) \tag{8}$$

$$P_y^c = L_2\sin(\theta_2) + r_2\sin\gamma_2 + A'P\sin(\theta_3^c + \beta) \tag{9}$$

Where P_x^c , P_y^c denote the $X\ and Y$ coordinate values for the path of the coupler point in considering the joint clearance.

In the kinematic analysis of the 4-bar mechanism with double joint clearance, it is necessary to determine the position of mass centre for moving links and then their corresponding velocities and accelerations. So in the case of joint clearance these positions are derived from the vector representation of the mechanism in Fig.3.

Due to motion transmission from crank link to follower, joint clearance between crank and coupler has important role on the path generation by coupler point P. If the crank pivot (A_1) is taken as the reference point, the mass centre positions for moving links are given as follows.

$$x_{G2}^{c} = A_0G_2 \cos\theta_2 \qquad (10)$$

$$y_{G2}^{c} = A_0G_2 \sin\theta_2 \qquad (11)$$

$$x_{G3}^{c} = L_2\cos\theta_2 + r_2\cos\gamma_2 + AG_3\cos(\theta_3^c + \delta) \qquad (12)$$

$$y_{G3}^{c} = L_2\sin\theta_2 + r_2\sin\gamma_2 + AG_3\sin(\theta_3^c + \delta) \qquad (13)$$

$$x_{G4}^{c} = L_2\cos\theta_2 + r_2\cos\gamma_2 + L_3\cos\theta_3^c + r_3\cos\gamma_3 + BG_4\cos\theta_3^c \quad (14)$$

$$y_{G4}^{c} = L_2\sin\theta_2 + r_2\sin\gamma_2 + L_3\sin\theta_3^c + r_3\sin\gamma_3 + BG_4\sin\theta_3^c \qquad (15)$$

4. Lagrange's equation

The Lagrange's equation is used to determine γ_2, γ_3 the direction of the mass less clearance link. For this specific purpose the equation can be expressed as

$$\frac{d}{dt}\left\{\frac{\partial}{\partial\dot{\gamma}_2}(K.E)\right\} - \frac{\partial}{\partial\gamma_2}(K.E) + \frac{\partial}{\partial\gamma_2}(P.E) + \frac{\partial}{\partial\dot{\gamma}_2}(D.F) = 0 \qquad (16)$$

Where, the kinetic energy ($K.E.$), potential energy ($P.E.$) and dissipation function ($D.F.$) are:

$$K.E. = \frac{1}{2}\sum_{i=2}^{4} I_i\left(\dot{\theta}_i^c\right)^2 + \frac{1}{2}\sum_{i=2}^{4} m_i\left[\left(\dot{x}_{Gi}^c\right)^2 + \left(\dot{y}_{Gi}^c\right)^2\right]$$

$$P.E. = \sum_{i=2}^{4} m_i g y_{Gi}^c \qquad (17)$$

$$D.F = \frac{1}{2}\sum_{i=2}^{4} C\theta_i\left(\dot{\theta}_i^c\right)^2 + \frac{1}{2}C_{\gamma_2}\dot{\gamma}_2^{\,2}$$

The Eqs. (17) together with Eq. (16) give,

$$\sum_{i=2}^{4}\left[I_i\ddot{\theta}_i^{\,c}\frac{\partial\theta_i^c}{\partial\gamma_2} + m_i\left(\ddot{x}_{Gi}^c\frac{\partial x_{Gi}^c}{\partial\gamma_2} + \ddot{y}_{Gi}^c\frac{\partial y_{Gi}^c}{\partial\gamma_2}\right) + gm_i\frac{\partial y_{Gi}^c}{\partial\gamma_2} + C_{\theta i}\dot{\theta}_i^c\frac{\partial\theta_i^c}{\partial\gamma_2} + C\gamma_2\dot{\gamma}_2\right] = 0 \qquad (18)$$

These differential equations determine the motion of the system. Similarly we can determine the direction of γ_3. They are used to determine the direction of the joint clearance with respect to input and are solved by DE on MATLAB software.

5. Case study

A case study is conducted four bar linkage with clearances in the revolute joint connecting the input link and the coupler. Values of dynamic parameters used in Lagrange's equation are given in Table 1.

Table .1. Assumed values of dynamic parameters

Descriptions	parameters	values
MOI of the link 2	I_2	$5.12\times10^{-4}\,Kg\ m^2$
MOI of the link 3	I_3	$8.85\times10^{-3}\,Kg\ m^2$
MOI of the link 4	I_4	$1.58\times10^{-3}\,Kg\ m^2$
Mass of the link2	m_2	$0.121 Kg$
Mass of the link 3	m_3	$1.048 Kg$
Mass of the link 4	m_4	$0.071 Kg$
Gravitational acceleration	g	$9.8\,m/s^2$
Input angular velocity	ω_2	$62.83\,rad/s$
Damping coefficient	$C_{\theta i}$	$0.2\times10^{-6}\,Kgms/rad$
Damping coefficient	$C_{\gamma 2}, C_{\gamma 3}$	$0.2\times10^{-6}\,Kgms/rad$

6. DE to determine direction of joint clearance

The equation of motion of the system has a non linear character as it is obvious from Eq. (18). The direction of the clearance link γ_2 and γ_3 is determined as a function of the direction of the input link by optimization approach using DE. The objective function is expressed as [12],

$$Minimize F(x) = f\left\{\sum_{i=2}^{4}\left[I_i\ddot{\theta}_i^c\frac{\partial\theta_i^c}{\partial\gamma_2} + m_i\left\langle\ddot{x}_{Gi}^c\frac{\partial x_{Gi}^c}{\partial\gamma_2} + \ddot{y}_{Gi}^c\frac{\partial y_{Gi}^c}{\partial\gamma_2}\right\rangle + gm_i\frac{\partial y_{Gi}^c}{\partial\gamma_2} + C_{\theta i}\theta_i^c\frac{\partial\theta_i^c}{\partial\gamma_2} + C_{\gamma_2}\dot{\gamma}_2\right]\right\},$$
$$h_j(x) = 0,\ x_l \le x_j \le x_u,\ x_j \in X, \qquad (19)$$

$$Minimize F(x) = f\left\{\sum_{i=2}^{4}\left[I_i\ddot{\theta}_i^c\frac{\partial\theta_i^c}{\partial\gamma_3} + m_i\left\langle\ddot{x}_{Gi}^c\frac{\partial x_{Gi}^c}{\partial\gamma_3} + \ddot{y}_{Gi}^c\frac{\partial y_{Gi}^c}{\partial\gamma_3}\right\rangle + gm_i\frac{\partial y_{Gi}^c}{\partial\gamma_3} + C_{\theta i}\theta_i^c\frac{\partial\theta_i^c}{\partial\gamma_3} + C_{\gamma_3}\dot{\gamma}_3\right]\right\},$$
$$h_j(x) = 0,\ x_l \le x_j \le x_u,\ x_j \in X, \qquad (20)$$

Where $h_j(x)$ are the equality constraints that depend on parametric relations between direction of clearance link and input variable; X is a vector that comprises the design variables corresponding to direction of clearance link, that is,

$\gamma_2, \dot{\gamma}_2$ and x_l and x_u are the lower and upper bounds of the design variables, respectively.

The objective function is the sum of two terms. The first part computes the position error (also called the structural error) as the sum of squares of the Euclidian distances between each desired points along the coupler curve $(Px_d{}^i, Py_d{}^i)$ that has to be traced by the mechanism and the corresponding generated points $(Px_g{}^i, Py_g{}^i)$ by the designed mechanism. The desired points are a set of target points along the coupler curve indicated by the designer and should be met by the coupler point of the mechanism and the generated curve is the curve that is actually obtained by the coupler point of the designed mechanism. For minimizing the error between desired and generated curves, the objective function is given by

$$Minimize\, F(x) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\left(Px_d^i - Px_g^i\right)^2 + \left(Py_d^i - Py_g^i\right)^2\right]}$$

$$subject\ to\ h_j(x) \le 0, x_l \le x_j \le x_u, \qquad (21)$$

$$where\ N = number\ of\ precision\ points$$

The inequality constraints $(h_j(x))$ consist of conditions of Grashof's rule that specifies crank rocker situation. x_l and x_u are respectively the lower and upper bounds of the design variables. They consist of link lengths L_i, $(Px_d{}^i, Py_d{}^i)$ and the structural angle, the angle that the line AP makes with the line of centre of the coupler AB as seen in Fig. 2 to obtain the co-ordinates of the generated positions of the coupler point displacement analysis is essential.

The successful application of the developed methodology for synthesis of mechanisms is shown through some case studies. All the data presented here are in a consistent set of units, i.e., all linear dimensions are in the unit of length and the angular dimensions are in degrees unless otherwise stated. For each case study an intermediate result and the final result are given. Finally, a summary of the results of all the cases indicating the values of the design variables obtained through DE is presented. The optimization algorithm is applied using MATLAB version 7.1. All the linkages shown in the following sections are obtained by using the design variables. Here the technique of geometric centroid of precision points (GCPP) [12] has been used to define the initial bounds of the design variables. The GCPP is obtained by evaluating the mean coordinates X_{cg} and Y_{cg} (mass center position) of the desired precision points in X and Y directions respectively.

7. Result considering different values of joint clearance.

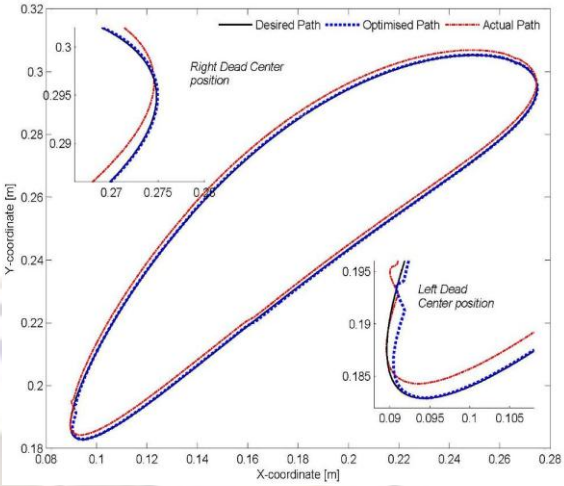

Fig 4. Coupler curve for r_2=1 mm and r_3=1 mm

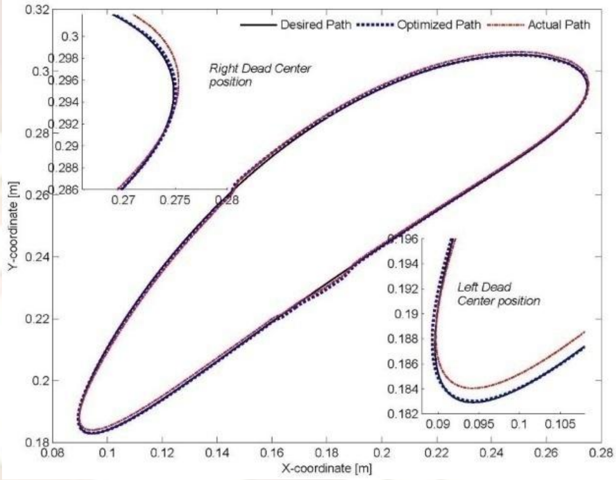

Fig .5. Coupler curve for r_2=1 mm and r_3=2 mm

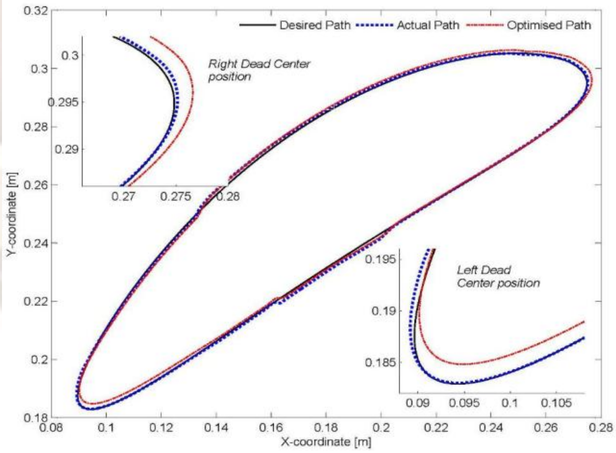

Fig.6. Coupler curve for r_2=2 mm and r_3=1 mm

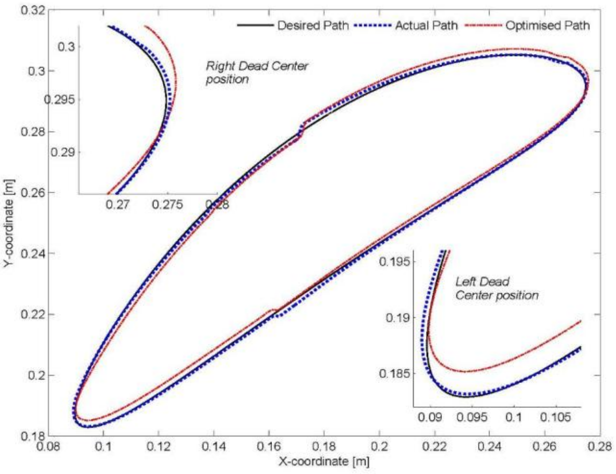

Fig.7. Coupler curve for r_2=2 mm and r_3=2 mm

Figs. (8-11) Direction of joint clearance with respect to position of input link (θ_2)

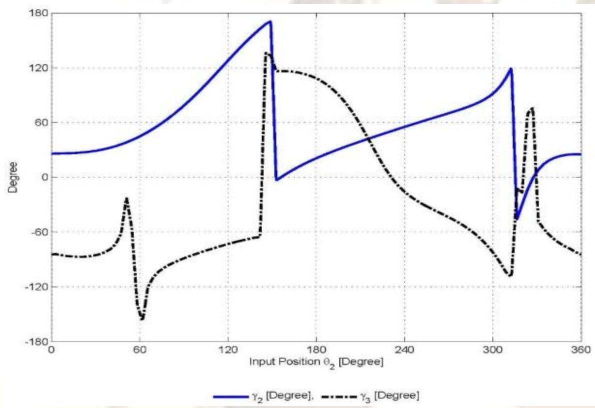

Fig.8. for r_2=1 mm and r_3=1 mm

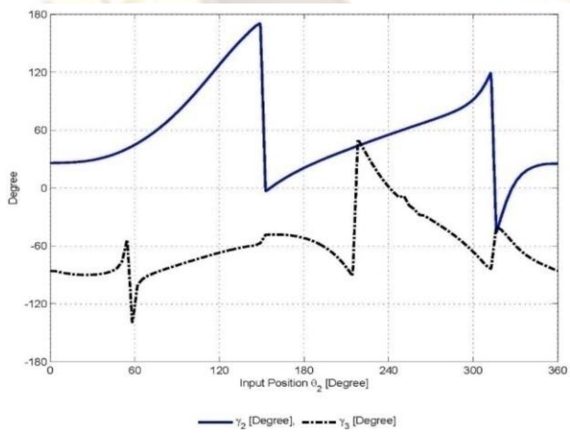

Fig.9 for r_2=1 mm and r_3=2 mm

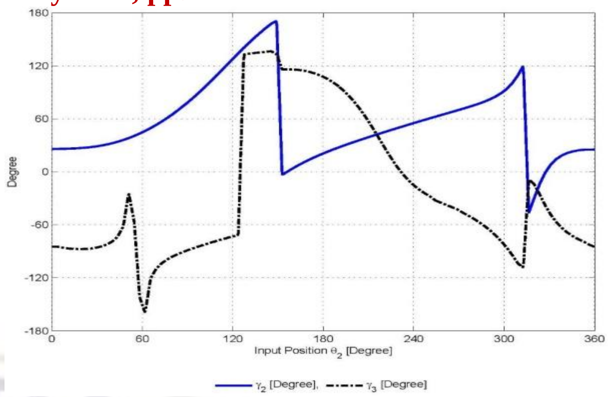

Fig.10 (θ_2) for r_2=2 mm and r_3=1 mm

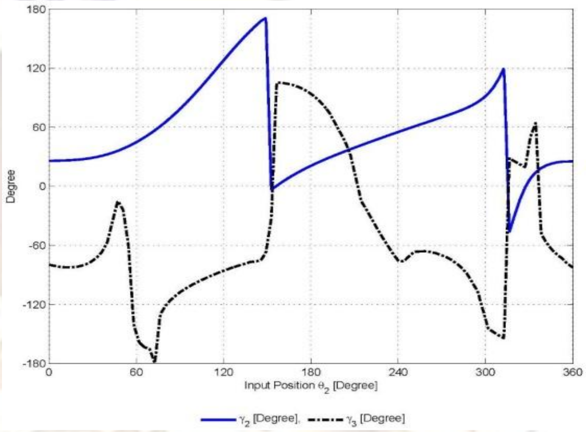

Fig.11. for r_2=2 mm and r_3=2 mm

Figs. (12-15) Path error between desired and actual mechanism and path error between desired and optimised mechanism.

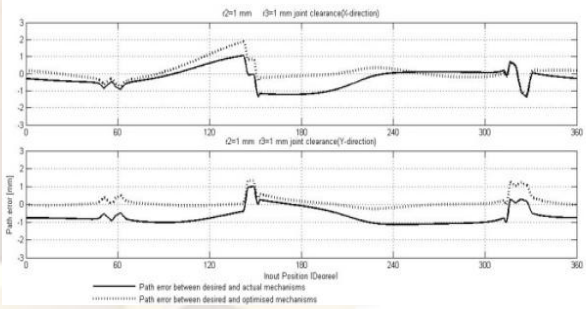

Fig.12 for r_2=1 mm and r_3=1 mm

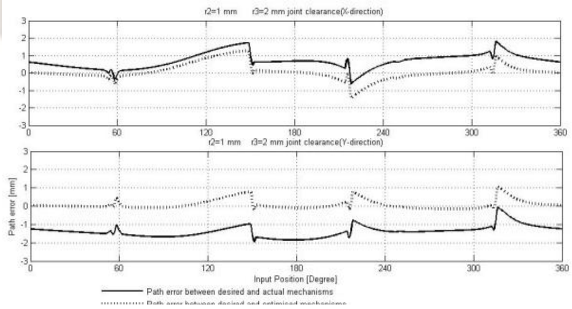

Fig.13 for r_2=1 mm and r_3=2 mm

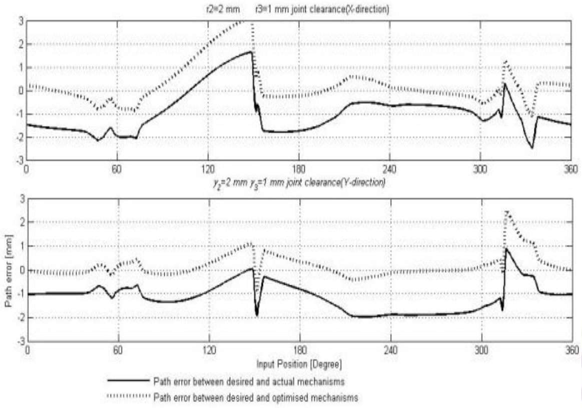

Fig.14. for r_2=2 mm and r_3=1 mm

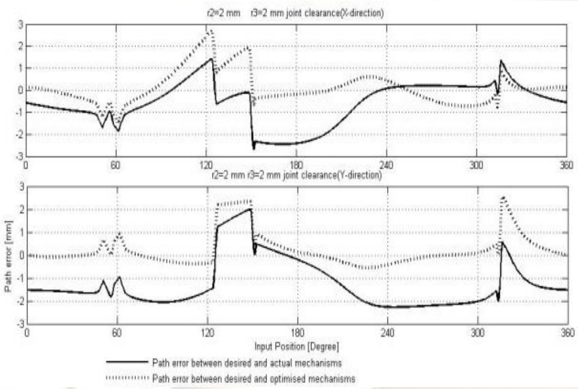

Fig.15 for r_2=2 mm and r_3=2 mm

7.1. Optimized design variables for different values of joint clearance

The minimized error between desired curve and generated curve are shown in table 2 for different clearance value.

Table 2 .Optimized design variables for different values of joint clearance

First clearance	1 mm	1 mm	2 mm	2 mm
Second clearance	1 mm	2 mm	1 mm	2 mm
L_1 (mm)	400.0481	399.8386	400.2295	401.1023
L_2 (mm)	100.0600	99.9970	100.1718	100.0828
L_3 (mm)	361.9587	359.2392	360.2949	365.5831
L_4 (mm)	249.2171	239.2488	247.5518	259.4261
A'P (mm)	304.1872	304.2217	303.3092	303.3529
β (degree)	18.7884	20.1741	19.1220	17.2177
Error (normalized) (mm)	0.369	0.332	0.635	0.700

8. Conclusion

In this study**,** for determining the effect of joint clearance on coupler curves the joints with clearance at the contact point of crank and coupler and contact point of coupler and follower have considered. Here the optimum link lengths are found out with an objective to minimize the error in the path generation considering the given joint clearances. It is observed from the result that after doing optimization we can reduce the error between the desired and optimized curve. These results can be seen as appropriate optimization technique for minimizing the actual error in the presence of clearances. The values of the clearances may vary due to precision to be maintained. Therefore optimum results are extracted for different set of values of the clearances to study the effect of joint clearances on the minimized error.

From the result it can be adapted to similar systems to improve the mechanical working conditions.

References

[1] R.H. Burns and F.R.E. Crossley, Structural Permutations of Flexible Link Mechanisms, ASME Paper, 66-Mech-5.

[2] T. G. Boronkay and C. Mei, Analysis and Design of Multiple Input Flexible Link Mechanisms, Journal of Mechanisms 5 (1970), pp.29-40.

[3] S. Dubowsky, On predicting the dynamic effect of clearances in planar mechanisms, ASME Journal of Engineering for industry 93B (1974) (1) pp. 317-323.

[4] T. Furuhashi, N. Morita and M. Matsuura, Research on dynamics of four-bar linkage with clearances at turning pairs (1st Report, General theory of continuous contact model), Bulletin of the JSME **21** (1978), pp. 518–523.

[5] N. Morita, T. Furuhashi and M. Matsuura, Research on dynamics of four-bar linkage with clearances at turning pairs (2nd Report, Analysis of crank-level mechanism with clearance at joint of crank and coupler using continuous contact model), Bulletin of the JSME **21** (1978), pp. 1284–1291.

[6] A.K. Mallik and S.G. Dhande, Analysis and synthesis of mechanical error in path-generating linkages using a stochastic approach, Mechanism and Machine Theory 22 (1987), pp. 115-123.

[7] K.W. Ting, J. Zhu and D. Watkins, The effects of joint clearance on position and orientation deviation of linkages and manipulators, Mechanism and Machine Theory **35** (2000), pp. 391–401.

[8] A.L. Schwab, J.P. Meijaard and P. Meijers, A comparison of revolute joint clearance models in the dynamic analysis of rigid and

elastic mechanical systems, Mechanism and Machine Theory 37 (2002) 895–913.
[9] M.J. Tsai and T.H. Lai, Kinematic sensitivity analysis of linkage with joint clearance based on transmission quality, Mechanism and Machine Theory **39** (2004), pp. 1189–1206.
[10] P. Flores, J. Ambrosio, J.C.P. Claro, H.M. Lankarani and C.S. Koshy, A study on dynamics of mechanical systems including joints with clearance and lubrication, Mechanism and Machine Theory 41 (2006), pp. 247–261.
[11] M.J. Tsai and T.H. Lai, Accuracy analysis of a multi-loop linkage with joint clearances, Mechanism and Machine Theory 43 (2008) 1141–1157.
[12] S. Erkaya and I. Uzmay, Determining link parameters using genetic algorithm in mechanisms with joint clearance, Mechanism and Machine Theory 44 (2009) 222–234.