FOURIER TRANSFORMS - APPROACH TO SCIENTIFIC PRINCIPLES

Edited by **Goran S. Nikolić**

INTECHWEB.ORG

Fourier Transforms - Approach to Scientific Principles
Edited by Goran S. Nikolić

Published by InTech
Janeza Trdine 9, 51000 Rijeka, Croatia

Publishing Process Manager Katarina Lovrecic
Technical Editor Teodora Smiljanic
Cover Designer Martina Sirotic
Image Copyright stevanovic.igor, 2010. Used under license from Shutterstock.com

First published March, 2011
Printed in India

A free online edition of this book is available at www.intechopen.com
Additional hard copies can be obtained from orders@intechweb.org

Fourier Transforms - Approach to Scientific Principles, Edited by Goran S. Nikolić
p. cm.
ISBN 978-953-307-231-9

Contents

6

Three Dimensional Reconstruction Strategies Using a Profilometrical Approach based on Fourier Transform

Pedraza-Ortega Jesus Carlos, Gorrostieta-Hurtado Efren, Aceves-Fernandez Marco Antonio, Sotomayor-Olmedo Artemio, Ramos-Arreguin Juan Manuel, Tovar-Arriaga Saul and Vargas-Soto Jose Emilio

Facultad de Informatica – Universidad Autonoma de Queretaro
Mexico

1. Introduction

In the past 3 decades, there is an idea to extract the 3D information of a scene from its 2D images ant it has been a research interest in many fields. The main idea is to extract the useful depth information from an image or set of images in an efficient and automatic way. The result of the process (depth information) can be used to guide various tasks such as synthetic aperture radar (SAR), magnetic resonance imaging (MRI), automatic inspection, reverse engineering, 3D robot navigation, interferometry and so on. The obtained information can be used to guide various processes such as robotic manipulation, automatic inspection, inverse engineering, 3D depth map for navigation and virtual reality applications (Gokstorp, 1995). Depending on the application, a simple 3D description is necessary to understand the scene and perform the desired task, while in other cases a dense map or detailed information of the object's shape is necessary. Furthermore, in some cases a complete 3D description of the object may be required.

In 3D machine vision, the three-dimensional shape can be obtained by using two different methodologies; Active and Passive Methods, which are also classified as contact and non contact methods. The active methods project energy in the scene and detect the reflected energy; some examples of these methods are sonar, laser ranging, fringe projection and structured method.

The fringe processing methods are widely used in non-destructive testing, optical metrology and 3D reconstruction systems. Some of the desired characteristics in these methods are high accuracy, noise-immunity and processing speed.

In the spatial and temporal fringe pattern analysis, the main characteristics are the number of fringes, and the intensity variation due temporal and spatial measurements.

A few commonly used fringe processing methods are well-known like Fourier Transform Profilometry (FTP) method (Malacara, 2006) and phase-shifting interferometry (Takeda et al., 1992). The main problem to overcome in these methods is the wrapped phase, where the depth information is included.

Despite the fact that most of the previous cited works are proved and tested in previous research, the present work present two strategies as an overview to the Fourier Transform Profilometry. The first one is a modified algorithm to the Fourier Transform Profilometry Method, where an additional pre-processing filter plus a data analysis in the unwrapping step is presented. The second strategy presented here is the use of the local and global phase unwrapping in the Modified Fourier Transform Profilometry. Both proposed methods present some advantages and the simplicity of the algorithm could be considered for implementation in real time 3D reconstruction.

2. Fourier transform profilometry

The image of a projected fringe pattern and an object with projected fringes can be represented by the following equations:

$$g(x,y) = a(x,y) + b(x,y) * \cos[2 * \pi f_0 x + \varphi(x,y)] \tag{1}$$

$$g_0(x,y) = a(x,y) + b(x,y) * \cos[2 * \pi f_0 x + \varphi_0(x,y)] \tag{2}$$

where $g(x,y)$ $g_0(x,y)$ are the intensity of the images at (x,y) point, $a(x,y)$ represents the background illumination, $b(x,y)$ is the contrast between the light and dark fringes, f_0 is the spatial-carrier frequency and $\varphi(x,y)$ and $\varphi_0(x,y)$ are the corresponding phase to the fringe and distorted fringe pattern, observed from the camera.
The phase $\varphi(x,y)$ contains the desired information, and $a(x,y)$ and $b(x,y)$ are unwanted irradiance variations. In most cases $\varphi(x,y)$, $a(x,y)$ and $b(x,y)$ vary slowly compared with the spatial-carrier frequency f0. Then, the angle $\varphi(x,y)$ is the phase shift caused by the object surface end the angle of projection, and its expressed as:

$$\varphi(x,y) = \varphi_0(x,y) + \varphi_z(x,y) \tag{3}$$

Where $\varphi_0\ (x,y)$ is the phase caused by the angle of projection corresponding to the reference plane, and φ_z(x,y) is the phase caused by the object's height distribution.
Considering the figure 1, we have a fringe which is projected from the projector, the fringe reaches the object at point H and will cross the reference plane at the point C. By observation, the triangles DpHDc and CHF are similar and

$$\frac{CD}{-h} = \frac{d_0}{l_0} \tag{4}$$

Leading us to the next equation:

$$\varphi_z(x,y) = \frac{h(x,y) 2\pi f_0 d_0}{h(x,y) - l_0} \tag{5}$$

Where the value of $h(x,y)$ is measured and considered as positive to the left side of the reference plane. The previous equation can be rearranged to express the height distribution as a function of the phase distribution:

$$h(x,y) = \frac{l_0 \phi_z(x,y)}{\phi_z(x,y) - 2\pi f_0 d_0} \tag{6}$$

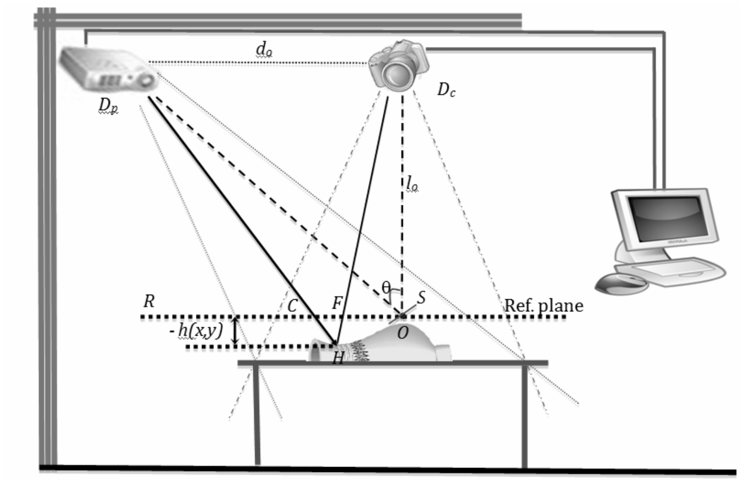


Fig. 1. Experimental setup

2.1 Fringe analysis

The fringe projection equation 1 can be rewritten as:

$$g(x,y) = \sum_{n=-\infty}^{\infty} A_n r(x,y) \exp(in\varphi(x,y)) * \exp(i2\pi n f_0 x) \tag{7}$$

Where r(x,y) is the reflectivity distribution on the diffuse object (Taketa et al., 1992) (Berryman et al., 2003). Then, a FFT (Fast Fourier Transform) is applied to the signal for in the x direction only. Notice that even y is considered as fix, the same procedure will be applied for the number of y lines in both images. Therefore, we obtain the next equation:

$$G(f,y) = \sum_{-\infty}^{\infty} Q_n(f - nf_0, y) \tag{8}$$

Now, we can observe that φ(x,y) and r(x,y) vary very slowly in comparison with the fringe spacing, then the Q peaks in the spectrum are separated each other. Also it is necessary to consider that if we choose a high spatial fringe pattern, the FFT will have a wider spacing among the frequencies. The next step is to remove all the signals with exception of the positive fundamental peak f_0. The obtained filtered image is then shifted by f_0 and centered. Later, the IFFT (Inverse Fast Fourier Transform) is applied in the *x* direction only, same as the FFT. The obtained equations for the reference and the object are given by:

$$\hat{g}(x,y) = A_1 r(x,y) \exp\{i(2\pi f_0 x + \varphi(x,y))\} \tag{9}$$

$$\hat{g}_0(x,y) = A_1 r_0(x,y) \exp\{i(2\pi f_0 x + \varphi_0(x,y))\} \tag{10}$$

By multiplying the $\varphi(x,y)$ with the conjugate of $\varphi_0(x,y)$, and separating the phase part of the result from the rest we obtain:

$$\begin{aligned}\varphi_z(x,y) &= \varphi(x,y)+\varphi_0(x,y)\\ &= \mathrm{Im}\{\log(\hat{g}(x,y)\hat{g}_0^*(x,y))\}\end{aligned} \tag{11}$$

From the above equation, we can see that the phase map can be obtained by applying the same process for each horizontal line. The values of the phase map are wrapped at some specific values. Those phase values range between π and -π.

To recover the true phase it is necessary to restore to the measured wrapped phase of an unknown multiple of $2\pi f_0$.The phase unwrapping process is not a trivial problem due to the presence of phase singularities (points in 2D, and lines in 3D) generated by local or global sub-sampling. The correct 2D branch cut lines and 3D branch cut surfaces should be placed where the gradient of the original phase distribution exceeded π rad value. However, this important information is lost due to undersampling and cannot be recovered from the sampled wrapped phase distribution alone. Also, is important to notice that finding a proper surface, or obtaining a minimal area or using a gradient on a wrapped phase will not work and could not find the correct branch in cut surfaces. From here, it can be observed that some additional information must be added in the branch cut placement algorithm.

Therefore, the next step is to apply some improved phase unwrapping algorithms. The whole methodology is described in figure 2.

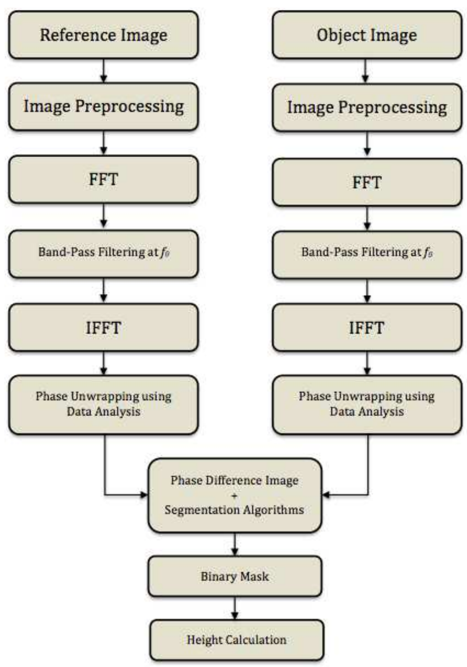


Fig. 2. Firstly Proposed Methodology

2.2 Phase unwrapping in the modified Fourier transform profilometry

As was early mentioned, the unwrapping step consists of finding discontinuities of magnitude close to 2π, and then depending on the phase change we can add or take 2π to the shape according to the sign of the phase change. There are various methods for doing the phase unwrapping, and the important thing to consider here is the abrupt phase changes in the neighbouring pixels. There are a number of 2π phase jumps between 2 successive wrapped phase values, and this number must be determined. This number depends on the spatial frequency of the fringe pattern projected at the beginning of the process.

This step is the modified part in the Fourier Transform Profilometry originally proposed by Takeda [3], and represents the major contribution of this work. Another thing to consider is to carry out a smoothing before the doing the phase unwrapping, this procedure will help to reduce the error produced by the unwanted jump variations in the wrapped phase map. Some similar methods are described in (Pramod, 2003)(Wu, 2006). Moreover, a modified Fourier Transform Profilometry method was used in (Pedraza et al, 2007) that include some extra analysis which considers local and global properties of the wrapped phase image.

Moreover, a second modification to the Fourier Transform Profilometry is proposed and presented in figure 3.

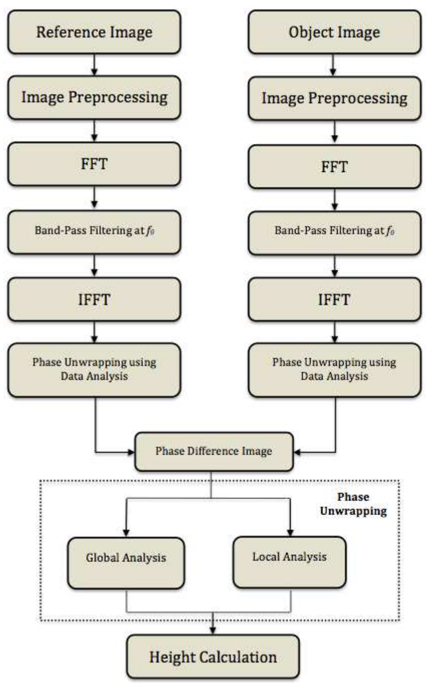


Fig. 3. Second Proposed Methodology

3. Phase unwrapping

Since two decades ago, phase unwrapping has been a research area and many papers have been published, presenting some ideas that solves the problem. Several phase unwrapping algorithms have been proposed, implemented and tested.

The phase unwrapping process is not a trivial problem due to the presence of phase singularities (points in 2D, and lines in 3D) generated by local or global undersampling. The correct 2D branch cut lines and 3D branch cut surfaces should be placed where the gradient of the original phase distribution exceeded π rad value. However, this important information is lost due to undersampling and cannot be recovered from the sampled wrapped phase distribution alone. Also, is important to notice that finding a proper surface, or obtaining a minimal area or using a gradient on a wrapped phase will not work and one could not find the correct branch in cut surfaces.

The phase unwrapping has many applications in applied optics that require an unwrapping process, and hence many phase unwrapping algorithms has been developed specifically for data with a particular application. Moreover, there is no universal phase unwrapping algorithm that can solve wrapped phase data from any application. Therefore, phase unwrapping algorithms are considered as a trade-off problem between accuracy of solution and computational requirements. However, even the most robust and complete phase unwrapping algorithm cannot guarantee in giving successful or acceptable unwrapped results without a good set of initial parameters. Unfortunately, there is no standard or technique to define the parameters that guarantee a good performance on phase unwrapping.

In literature, exist several phase unwrapping algorithms, a general review of the most widely used algorithms used started with the single phase unwrapping algorithm proposed by (Takeda et al, 1982), later the continuous phase map was proposed by (Giglia & Pritt, 1998) and more recently (Kian et al., 2005) proposed a windowed fourier transform as a filter to approach the phase unwrapping, later the local and global analysis was proposed by (Pedraza et al, 2007). Broadly speaking, the local phase unwrapping algorithms can be divided in two main subcategories *quality guided and residue balancing or branch cuts.* On the other hand are the global phase unwrapping algorithms that deal with the problem of phase unwrapping in a minimum-norm (or minimization) approach, example of this phase unwrapping algorithms are unweighted least squares, weighted least squares, etc.

Generally, in order to face the phase unwrapping problems, algorithms can be divided in two categories: local and global phase unwrapping. Local phase unwrapping algorithms find the unwrapped phase values by integrating the phase along a certain path. This is called path-following algorithms. Another way to classify the phase unwrapping algorithms are temporal (mention some algorithms) or spatial (add some algorithms too) phas unwrapping according with the appropriate fringe pattern analysis. The post processing of the unwrapped phase is needed in order to improve the 3D Reconstruction results, and some analysis that were carried out by (Kian Q, 2007).

Global phase unwrapping algorithms locate the unwrapped phase by minimizing a global error function and are also called local phase unwrapping algorithm and a global phase unwrapping algorithm, by following the methodology proposed by us in (Pedraza et al., 2009). The unwrapped phase values and the wrapped phase can be related with each other according with the Shannon's sampling theorem:

$$\Psi(n) = \varphi(\pi) + 2\pi\, k(n) \qquad -\pi < \Psi(n) \le \pi \tag{12}$$

$$\varphi(n) = \Psi(n) + 2\pi\, v(n) \qquad -\infty < \varphi(n) \le \infty \tag{13}$$

here $\Psi(n)$ holds the wrapped phase values and $\varphi(n)$ holds the unwrapped phase values, $k(n)$ is the function containing the integers that must be added to the wrapped phase φ to be

unwrapped, n is an integer and *v(n)* is the function containing a set of integers that must be added to the wrapped phase Ψ.
Noticing that;

$$v(n) = -k(n) \tag{14}$$

The wrapping operation ω which converts the unwrapped phase is defined by:

$$\varpi\{\varphi(n)\} = \arctan\left[\frac{\sin(\varphi(n))}{\cos(\varphi(n))}\right] \tag{15}$$

3.1 Local phase unwrapping

Local phase unwrapping algorithms finds the unwrapped phase values by integrating the phase along certain path that covers the whole wrapped phase map. The local phase unwrapping defines the quality of each pixel in the phase map to unwrap the highest quality pixels first and the lowest quality pixels last (quality- guided phase unwrapping). The second type is known as residue-balancing methods, which attempts to prevent error propagation by identifying residues (the source of noise in the wrapped phase). The residues must be balanced and isolated by using barriers (branch-cuts), therefore, it aims to produce a path-independent wrapped phase map. Path-dependency occurs to the existence of residues.

Residue-balancing algorithms search for residues in a wrapped-phase map and attempt to balance positive and negative residues by placing cut lines between them to prevent the unwrapping path breaking the mesh created. The residue is identified for each pixel in the phase map by estimating the wrapped gradients in a 2 × 2 closed loop, as shown in Figure 4.

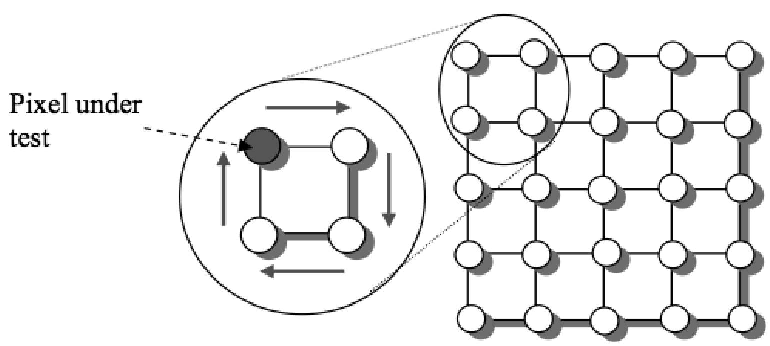


Fig. 4. Identifying residues in a 2 × 2 closed path

This is carried out using the following equation:

$$r = \Re\left[\frac{\Psi_{i,j} - \Psi_{i+1,j}}{2\pi}\right] + \Re\left[\frac{\Psi_{i+1,j} - \Psi_{i+1,j+1}}{2\pi}\right] + \Re\left[\frac{\Psi_{i,j+1} - \Psi_{i,j}}{2\pi}\right] \tag{26}$$

Where **$\Re$[]** rounds its argument to the nearest integer, *Ψx,y* is the wrapped pixel. The equation of interest; $f(x) = a(x) + b(x)\cos(2\pi f_0 x + \varphi(x))$ can only take three possible results: 0,

+1, and -1. A pixel under test is considered to be a positive residue if the value of r is +1, and it is considered to be a negative residue if the value is -1. Conversely, the pixel is not a residue if the value of *r* is zero. After identifying all residues in the wrapped phase map, these residues have to be balanced by means of branch cuts. Branch-cuts act as barriers to prevent the unwrapping path going thorough them. If these branch cuts are avoided during the unwrapping process, no errors propagate and the unwrapping path is considered to be path independent. On the other hand, if these branch cuts are penetrated during the unwrapping, errors propagate throughout the whole phase map, and in this case the unwrapping path is considered to be path dependent.

3.2 Global phase unwrapping

In the previous section, it was stated that local phase unwrapping algorithms follow a certain unwrapping path in order to unwrap the phase. They begin at a grid point and integrate the wrapped phase differences over that path, which ultimately covers the entire phase map. Local phase unwrapping algorithms (residue-balancing algorithms) generate branch cuts and define the unwrapping path around these cuts in order to minimize error propagation.

In contrast, global phase unwrapping algorithms formulate the phase unwrapping problem in a generalized minimum-norm sense (Ichioka & Inuiya, 1972). Global phase unwrapping algorithms attempt to find the unwrapped phase by minimizing the global error function as:

$$\varepsilon^2 = \|\text{solution} - \text{problem}\|^2 \tag{17}$$

Global phase unwrapping algorithms seek the unwrapped phase whose local gradients in the x and y direction match, as closely as possible.

$$\varepsilon^2 = \sum_{i=0}^{M-2}\sum_{j=0}^{N-1}\left|\Delta^x\varphi(i,j) - \hat{\Delta}^x\psi(i,j)\right|^p + \sum_{i=0}^{M-1}\sum_{j=0}^{N-2}\left|\Delta^y\varphi(i,j) - \hat{\Delta}^y\psi(i,j)\right|^p \tag{18}$$

Where $\Delta^x\varphi(i,j)$ and $\Delta^y\varphi(i,j)$ are unwrapped phase gradients in the x and y directions respectively, which are given by:

$$\Delta^x\varphi(i,j) = \varphi(i+1,j) - \varphi(i,j) \tag{19}$$

$$\Delta^y\varphi(i,j) = \varphi(i,j+1) - \varphi(i,j) \tag{20}$$

$\hat{\Delta}^x\psi(i,j)$ and $\hat{\Delta}^y\psi(i,j)$ are the wrapped values of the phase gradients in the x and y directions respectively, and they are given by:

$$\hat{\Delta}^x\psi(i,j) = \omega\{\psi(i+1,j) - \psi(i,j)\} \tag{21}$$

$$\hat{\Delta}^y\psi(i,j) = \omega\{\psi(i,j+1) - \psi(i,j)\} \tag{22}$$

Finally the wrapping operator is defined by the equation 15.

4. Experimental results

An experimental setup such as the one shown on figure 1 is suitable to apply the proposed methodology. In figure 1, a high resolution digital projector is used to create the structured light fringe pattern, and a mega-pixel digital CCD camera is used as a sensor to acquire the images. Also, a high-resolution digital CCD camera can be used instead of the mega-pixel camera. The reference plane can be any flat surface like a plain wall, or a whiteboard. In the reference plane is important to notice that the surface is a non-reflecting one in order to avoid the unwanted reflections that may affect or distort the image acquisition process. The object of interest can be any object and for this project, 2 objects are considered; the first one is an oval with a symmetrical shape and also a pyramid.

To create different fringe patterns, a GUI was developed. The GUI is capable to create several patterns by modifying the spatial frequency (number of fringes per unit area), and resolution (number of levels to create the sinusoidal pattern) of the fringe pattern. The GUI has the capability to do phase shifting if necessary and also the projection of the fringe pattern can have a horizontal or vertical orientation.

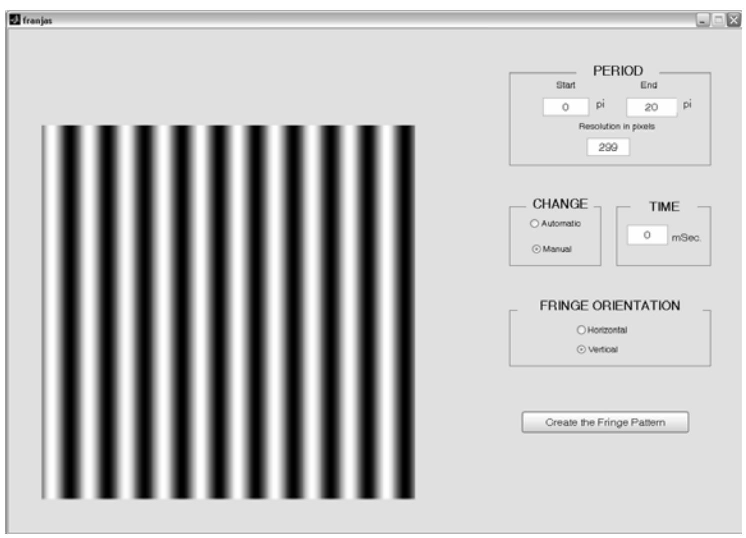


Fig. 3. Fringe Pattern GUI in MATLAB

As an example of one object, we can see on figure 4 the reference pattern projected on a plane and the same pattern projected on the object.

Applying the modified Fourier Transform Profilometry we can obtain the Fourier spectra corresponding to the images on figure 5.

On the left part of the figure 6 we can observe the wrapped depth map before applying the unwrapped algorithm. Usually, in doing phase unwrapping, linear methods are used [5-7]. These methods fail due to the fact the in the wrapped direction of the phase, a high frequency can be present and a simple unwrapping algorithm can generate errors in the mentioned direction. That is the main reason why a more complete analysis should be performed. In this research, a local discontinuity analysis together with the use of the global analysis is also implemented.

The main algorithm for the local discontinuity analysis [8] is described as; a) first, divide the wrapped phase map in regions and give a different weights (w1, w2, .., wn) to each region, b) the modulation unit is defined and helps to detect the fringe quality and divides the fringes into regions, c) regions are grouped from the biggest to the smallest modulation value, d) next, the unwrapping process is started from the biggest to the smallest region, e) later, an evaluation of the phase changes can be carried out to avoid variations smaller than f0.

After the local analysis, an unwrapping algorithm is applied considering punctual variations in the phase difference image, which will lead us to the desired phase distribution profile, that is, the object's form. Also, to help the selection of the proper value, a binary mask is used, like the one showed on figure 6 on the right hand. This binary mask gives an extra parameter, which has the value of "1" in the pixel where a phase jump is bigger than 2π. From the figure 6, it can be shown that there is more than 1 jump in a frequency higher than f0.

All the phase unwrapping was carried out in the y direction.

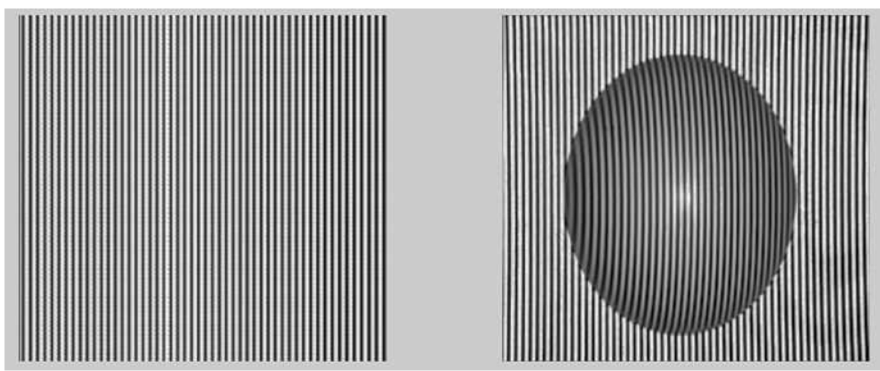


Fig. 4. Fringe Pattern projected on a plane and object to digitize respectively

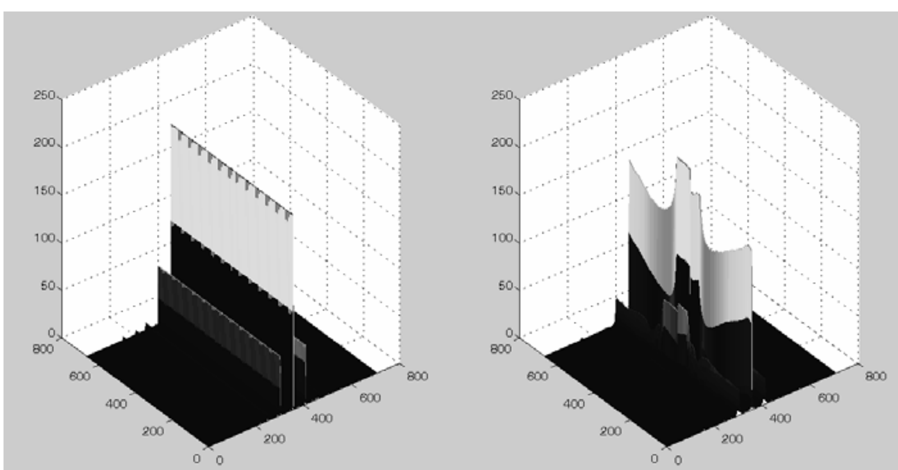


Fig. 5. Phase of projected fringe pattern on a plane and object to digitize respectively

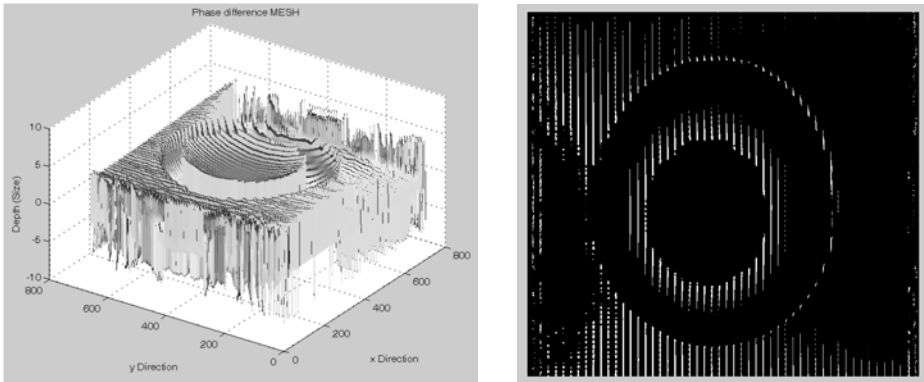


Fig. 6. Wrapped mesh and binary mask

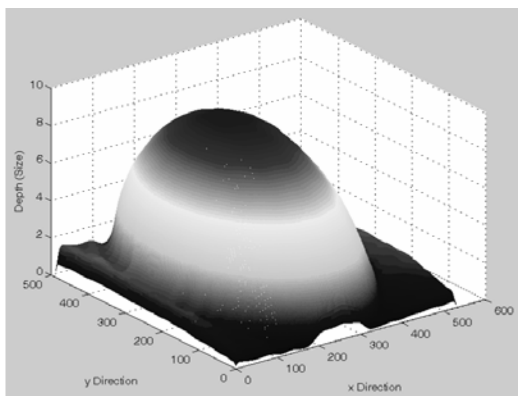


Fig. 7. The final 3D reconstruction mesh after the proposed methodology was applied

On figure 7, the final 3D reconstruction mesh is obtained after the proposed methodology is applied.

For the experimentation, a CCD camera SONY TRV-30 1.3 Mega-pixels was used. As a reference frame, a wooden made plane was used, and it was painted with black opaque paint to avoid glare. The digitized objects were an oval and a pyramid.

On Figure 8, a pyramid, the second object used in this work is presented. The projected fringe pattern and its Fourier spectra is observed, where a nearly 50 pixel spatial frequency is observed. Figure 9 shows the wrapped phase of the pyramid, and after applying the phase difference algorithm and binary mask, the 3D mesh of the object is presented.

As a second test, some computer based simulations using virtual created objects were carried out. On figure 10, a computer created hand was created. In this figure, the mesh visualization, as well as the projected pattern are presented Later, on figure 11, the wrapped phase image as well as the phase mesh are presented. Finally, after applying the proposed strategy, the reconstructed object is presented by using the global phase unwrapping and the local phase unwrapping. Two more object

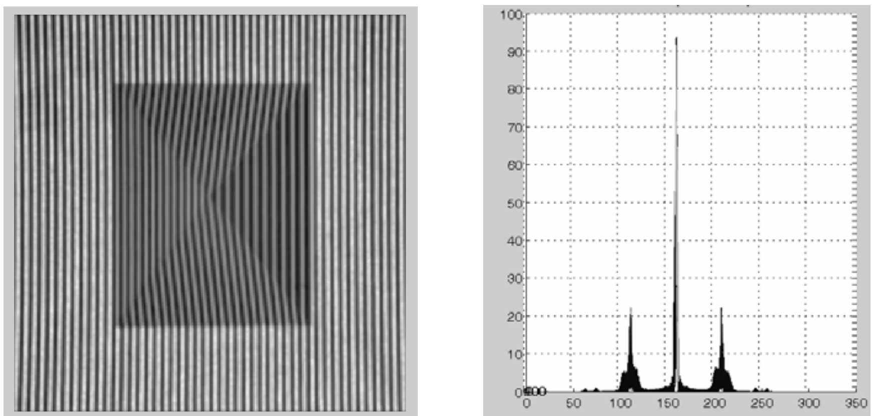


Fig. 8. Fringe pattern projected on a pyramid object and its phase

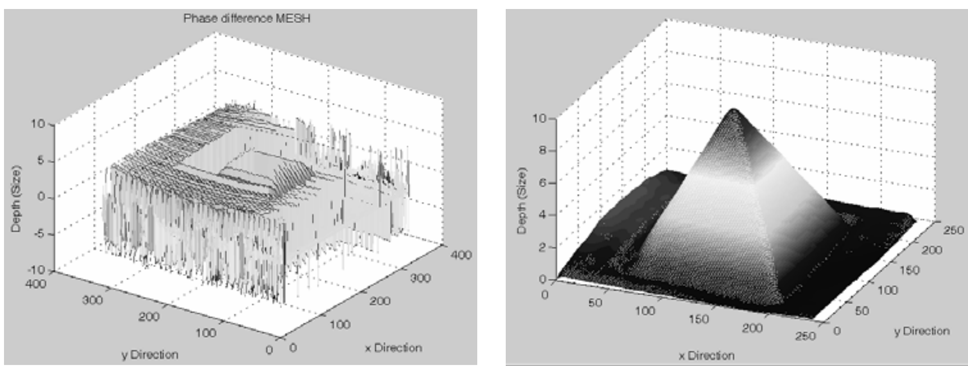
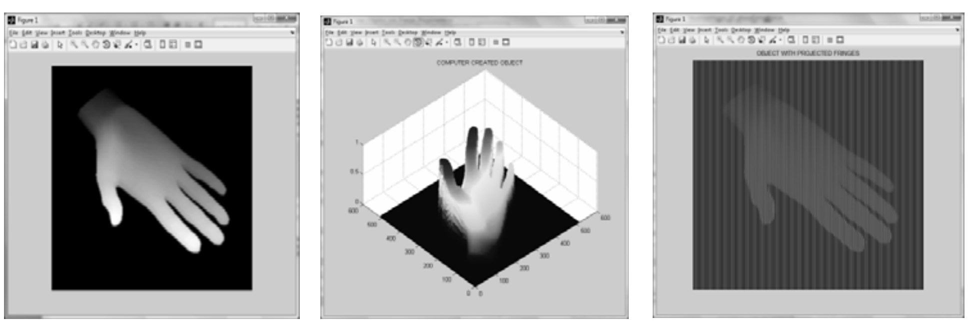
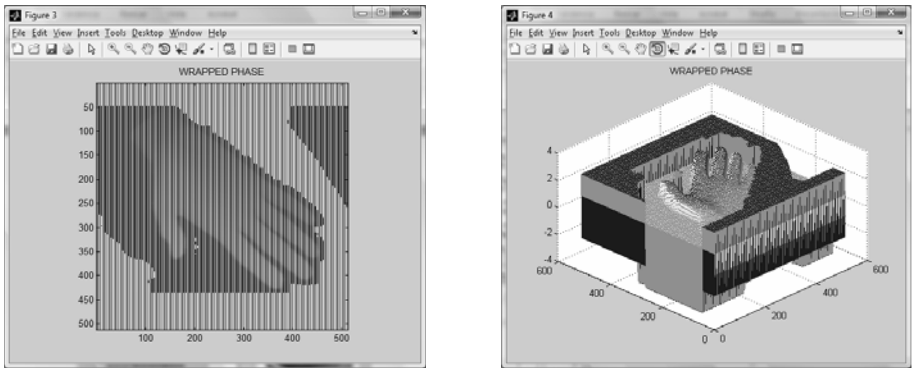


Fig. 9. Pyramid wrapped phase and its 3D reconstruction after the proposed method is applied



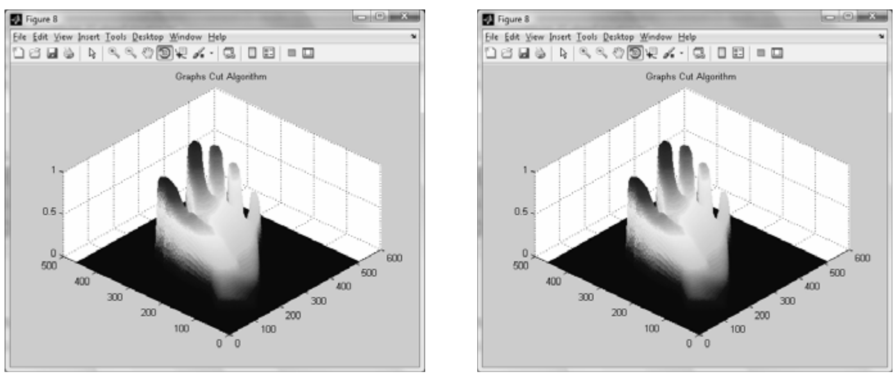
(a) (b) (c)

Fig. 10. Computer created object: (a) Hand, (b) Mesh Visualization and (c) Projected fringes



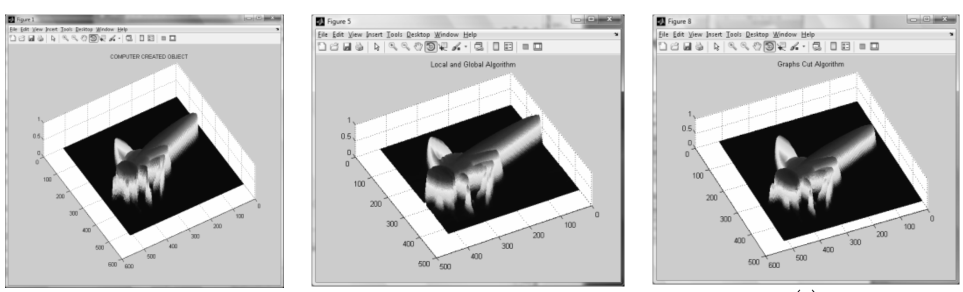
(a) (b)

Fig. 11. Computer created object: (a) Wrapped phase image, (b) Wrapped phase Mesh



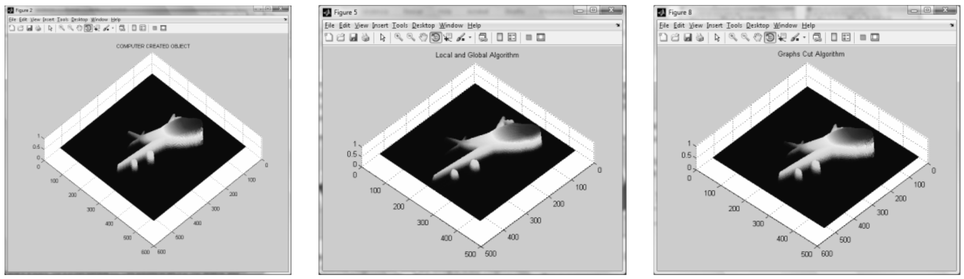
(a) (b)

Fig. 12. After applying the second strategy: (a) using local phase unwrapping, (b) using global phase unwrapping



(a) (b) (c)

Fig. 13. Dragonfly digitizing using the Second Strategy: (a) Virtual object to digitize (b) using local phase unwrapping, (c) using global phase unwrapping



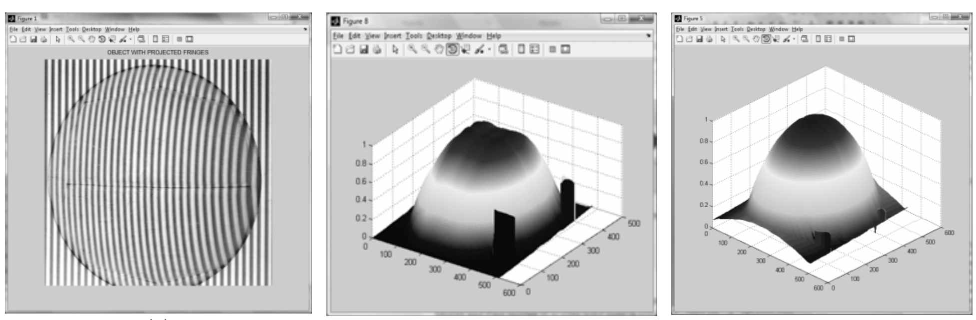
(a) (b) (c)

Fig. 13. Plane digitizing using the Second Strategy: (a) Virtual object to digitize (b) using local phase unwrapping, (c) using global phase unwrapping

Object	Local	Global
Hand	4.45	3.51
Dragonfly	4.67	3.81
Airplane	4.58	3.67

Table 1. Comparison between the phase unwrapping algorithms in the second strategy

Finally, the methodology was applied to a real object, which is presented on figure 14(a). The object is a volley ball, and the results of the local and global phase unwrapping algorithms are presented on the same figure 14(b) and (c) respectively.



(a) (b) (c)

Fig. 13. Object to digitize: (a) Volley ball (a) using local phase unwrapping, (b) using global phase unwrapping

5. Conclusions and future work

There are several methods and techniques to made three-dimensional reconstruction of virtual and real objects. Among all these methods the fringe pattern analysis had been widely used since it provides a non-destructive approach, to optical metrology and 3D reconstruction systems.

As an option to get the 3D reconstruction of objects, two basic strategies were introduced. A modified Fourier Transform Profilometry methodology and the Modified Fourier Transform Profilometry including the local and global phase unwrapping were described.

The fringe pattern analysis has two main phases, the phase extraction and the phase unwrapping; in this chapter a two modified profilometry methodologies are presented in order to perform this two phases.

The phase extraction basically consists in analyse a distorted fringe pattern image by using Fourier transform and filtering the undesired noise and frequency. The result of this phase is commonly a phase map wrapped into π and $-\pi$ range. Therefore a phase unwrapping algorithm is applied to recover the accurate phase map from the wrapped phase map. In literature the phase unwrapping algorithms have been classified in local and global phase unwrapping algorithms.

These phase unwrapping algorithms have been reviewed in this chapter. Local phase unwrapping algorithms are reasonably swift and had low computational requirements although it implies a decreasing in the quality of three-dimensional reconstruction. Conversely the global phase unwrapping algorithms are high time-consuming and had elevated computational requirements and commonly perform a superior quality of 3-D reconstruction.

This methodology could be widely used to digitize diverse objects for reverse engineering, virtual reality, 3D navigation, and so on.

Notice that the method can reconstruct only the part of the object that can be seen by the camera, if a full 3D reconstruction (360 degrees) is needed, a rotating table is can be used and the methodology will be applied n times, where n is the rotation angle of the table.

One big challenge is to obtain the 3D reconstruction in real time. As a part of the solution, an optical filter to obtain the FFT directly can be used. Moreover, the algorithm can be implemented into a FPGA to carry out a parallel processing and minimize the processing time. Some other tools include the testing of the algorithm performance and doing a comparison with a wavelet or neural networks.

6. References

Berryman, F; Pynsent, P.; Cubillo, J.; A theoretical Comparison of three fringe analysis methods for determining the three-dimensional shape of an object in the presence of noise, Optics and Lasers in Engineering,Vol. 39, pp. 35-50, 2003, ISSN 0143-8166 4

Giglia, D.C.; Prittm M.D., *Two dimensional phase unwrapping theory, algorithm and software*, Edit. Wiley, New York, 1998, ISBN 9780471249351.

Gokstorp, M.(1995). *Depth Computation in Robot Vision*, Ph.D. Thesis, Department of Electrical Engineering, Linkoping University, Linkoping, Sweden.

Ichioka, Y.; Inuiya, M.; Direct Phase Detecting System, *Applied Optics*, Vol. 11, Issue 7, pp. 1507-1514 , 1972, ISSN 1559-128X.

Itoh, K.; Analysis of the phase unwrapping algorithm, *Applied Optics*, 21(14): 2470-2486, 1982, ISSN 1559-128X.

Kian, Q.; Soon, S.H.; Asundi, A.; A simple phase unwrapping approach based on filtering by windowed Fourier transform, *Optics & Laser Technology*, Vol. 37, Issue 6, September 2005, pp. 458-462, ISSN 0030-3992.

Kian, Q.; Two dimensional windowed Fourier transform for fringe pattern analysis: Principles, applications and implementations, *Optics & Laser in Engineering*, Vol. 45, pp. 304-317, 2007, ISSN 0143-8166.

Malacara, D.; *Optical Shop Testing*, D Malacara, Ed. Wiley, New York, 2006.

Pedraza, J.C.; Rodriguez W.; Barriga, L.; Ramos, J.; Gorrostieta, E.; Rivas, A.; Image Processing for 3D Reconstruction using a Modified Fourier Transform Profilometry Method, *Lecture Notes on Artificial Intelligence (Advances in Artificial Intelligence)*, pp. 705–712, Springer-Verlag Berlin Heidelberg, 2007, ISSN 0302-9743.

Pedraza, J.C.; Canchola S.L.; Gorrostieta E.; Aceves M.A.; Ramos, J.M.; Delgado M.: Three-Dimensional Reconstruction System based on a Segmentation Algorithms and a Modified Fourier Transform Profilometry, *Proceedings of the IEEE Electronics, Robotics and Automotive Mechanics Conference CERMA 2009*, Morelos Mexico, 2009., ISBN 9780769537993.

Pedraza, J.C.; Gorrostieta E.; Ramos, J.M.; Canchola S.L.; Aceves M.A.; Delgado M.; Rico, R.A., A Profilometric Approach for 3D Reconstruction Using Fourier and Wavelet Transforms, *Lecture Notes on Artificial Intelligence (Advances in Artificial Intelligence)*, pp. 313-323, 2009, ISSN 0302-9743.

Pedraza, J.C.; Gorrostieta E.; Delgado M.; Canchola, S.L.; Ramos, J.M.; Aceves, M.A., Sotomayor, A.; A 3D Sensor Based on a Profilometric Approach, *Sensors Journal (www.mdpi.com/journal/sensors)*, pp. 10326-10340, ISSN 1424-8220,2009. 11Pramod, K; Digital Speckle Pattern Interferometry and related Techniques. Edit. Wiley, 2001, ISBN 9780471490524 5.

Sotomayor A.; Pedraza, J.C; Aceves, M.A.; Gorrostieta E.; Canchola, S.L.; Ramos, J.M.; Qintanar, M.E.; (2010). A Comparison between Local and Global Phase Unwrapping Algorithms in a Modified Fourier Transform Profilometry Method, *Proceedings of the IEEE, 20th International Conference on Electronics, communications and computers CONIELECOMP 2010,* pp.301-306, ISBN 978-1-4244-5353-5-10, Cholula Puebla Mexico, February 2010. IEEE, Puebla.

Takeda, M; Ina, H. Kobayashi, S; Fourier-Transform method of fringe pattern analysis for computed-based topography and interferometry. J.Opt. Soc.Am. Vol. 72, No.1, pp. 156-160, January 1982, ISSN 1084-7529. 3

Wu, L.; Research and development of fringe projection-based methods in 3D shape reconstruction, *Journal of Zhejiang University SCIENCE A*, pp. 1026-1036, 2006, ISSN 1673-565X 7